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Output Adaptive Compensation of External Disturbances in MIMO Systems

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Abstract—A problem of output-feedback adaptive compensation of external unknown deterministic disturbances in linear multidimensional plants is considered. The proposed solution is based on an adaptive implementation of the internal model principle together with a special observer. The form of the observer allows us to obtain a regression model of the multidimensional disturbance and design an adaptive controller with a number of tuning parameters equal to the number of unknown coefficients of the characteristic polynomial of the disturbance model. Under certain conditions, the dynamic order of the observer is significantly lower than that of well-known solutions.

Keywords: adaptive compensation of external disturbances, internal model principle, multidimensional systems

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1. INTRODUCTION

The internal model principle is one of an effective and well developed methods for asymptotic compensation of external deterministic disturbances due to many practical applications such as active vibration systems [1–3], internal combustion engines [4, 5], rolling mills [6, 7], continuous casting process [8], computer hard disk drives [9, 10], seismic protection systems [11], offshore cranes [12, 13], systems for ripple reduction in current converters [14] and in synchronous machines [15], etc. According to this principle, an external disturbance is modelled as the output of an autonomous dynamic system (exosystem), and this exosystem is replicated in the structure of the closed-loop system for the disturbance compensation. Initially, the internal model principle was developed for the linear systems and the linear model of external disturbances with known parameters [16; 17; 18, Chapter 4]. However, the assumption about the knowledge of the disturbance model parameters reduces the practical relevance of the method. The case when only the class of disturbances is known in a priori, but not the parameters of a disturbance model, is of great practical significance.

An adaptive version of the internal model principle, when the parameters of an exosystem model are unknown (see [19, Section 1.4; 20] and the references therein) was suggested for the compensation of unknown disturbances. This approach was developed for both systems with the scalar input and the scalar output [21–25] as well as for multidimensional systems with the state-feedback control [19, Section 4.2; 26; 27] or with the output-feedback [20, 28]. A special class is represented by multidimensional systems with the input disturbance [29].

However, well-known solutions of the output-feedback disturbances compensation in multidimensional systems using the internal model principle (both non-adaptive [17; 18, Chapter 4] and adaptive [20, 28]) include construction of a special observer (dynamic filter, precompensator) for each channel of a multidimensional system. The dimension of this observer is equal to the dimension of the disturbances generator model. Therefore, the total dimension of the used observers is equal to the product of the dimension of disturbances generator model by the number of channels in a control system. It causes increasing the dynamic order of the controller as well as of the number of adjustable parameters.

It is shown in the paper that under additional nonrestrictive assumption about the block-diagonal structure of the disturbance generator it is possible to design a control system with one observer of dimension equaled to dimension of the disturbance generator. As a result, both dynamic order of controller and the number of adjustable parameters are reduced significantly.

The solution proposed is based on the approach reported in [19, 30, 31] and consists in construction of a special external disturbance observer that makes it possible to get the convenient unmixed parametrization of a disturbance.

The paper is organized as follows. In Section 2, the problem statement is described. In Section 3, a disturbance observer is presented, and the problems of its construction for multidimensional systems are discussed. In Section 4 and 5, an adaptive controller is designed. An example with the simulation results is presented in Section 6.

Notations: $|y|$ is the Euclidean norm of a vector y; s is a complex variable; $W(s)[\delta]$ denotes a signal transformation δ (scalar or vector signal) by dynamic block with the transfer function (or the transfer matrix for a vector signal) $W(s)$.

2. PROBLEM STATEMENT

We consider the multidimensional time-invariant plant

$$\begin{cases} \dot{x} = Ax + B(u+\delta), \quad x(0), \\ y = Cx, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the vector of control signals, $y \in \mathbb{R}^m$ is the vector of output variables, $n \geqslant m$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$ are known matrices with constant entries, $\delta \in \mathbb{R}^m$ is the vector of unmeasured external disturbances.

Assumption 1. The following assumptions are accepted regarding the plant (1):

A.1.1. the triplet (A, B, C) is fully controllable and observable, and matrices B and C have rank, i.e. $\operatorname{rank} B = \operatorname{rank} C = m$;

A.1.2. the state-space model (1) is minimum phase;

A.1.3. the vector of output variables y is measurable only, while the state vector x is not accessible for measurements.

Remark 1. Assumption A.1.1 implies that the system (1) is controllable with regard to the vector of the output variables y (see the definition and the criteria in [32]).

Remark 2. In this paper, the minimum phase model (1) is the model that does not have invariant zeros or all its invariant zeros have negative real parts. An *invariant zero* is a complex number s_0, which substitution into the Rosenbrock matrix (that is, for $s = s_0$)

$$P(s) = \begin{bmatrix} sI - A & B \\ -C & 0 \end{bmatrix}$$

reduces the column rank of the matrix [33, p. 237]. For systems with the same number of control signals and the output variables having square Rosenbrock matrix, it means singularity of this matrix for $s = s_0$.

Assumption 2. The external disturbance $\delta = [\delta_1, \ldots, \delta_m]^\top$ is such that:

A.2.1. each its component δ_i $(i = 1, \ldots, m)$ can be represented as the output of the linear generator

$$\begin{cases} \dot{\xi}_i = \Gamma_i \xi_i, & \xi_i(0), \\ \delta_i = h_i^\top \xi_i, \end{cases} \tag{2}$$

where $\xi_i \in \mathbb{R}^{q_i}$ is the state vector, $\Gamma_i \in \mathbb{R}^{q_i \times q_i}$ is the matrix with constant parameters, all eigenvalues of which are on the imaginary axis and simple, $h_i \in \mathbb{R}^{q_i}$ is the vector of constant parameters;

A.2.3. parameters of the matrices Γ_i and the vectors h_i are unknown, while the dimensions q_i are known;

A.2.3. the pairs $(h_i^\top, \Gamma_i)$ are fully observable.

Remark 3. The general exosystem generating the disturbance δ can be represented in the form

$$\begin{cases} \dot{\xi} = \Gamma \xi, & \xi(0), \\ \delta = H \xi, \end{cases} \tag{3}$$

where $\xi = [\xi_1^\top, \ldots, \xi_m^\top]^\top \in \mathbb{R}^q$, matrices $\Gamma = \operatorname{diag}\{\Gamma_i\}$ and $H = \operatorname{diag}\{h_i^\top\}$ $(i = 1, \ldots, m)$ are block-diagonal, and $q = \sum_{i=1}^m q_i$.

The following problem is solved in the paper.

Problem 1. The problem is to design an output-feedback control law providing the boundedness of all the closed-loop signals and the achievement of the limiting equality

$$\lim_{t \to \infty} |y(t)| = 0. \tag{4}$$

In other words, the plant stabilization together with the external disturbance compensation is required simultaneously. Since according to Assumption A.2.3 the external disturbance is unknown a priori, then the classical internal model principle [16–18] is not applicable. Therefore, in order to solve the problem, an adaptive modification of the internal model principle [19, 21, 31] will be used. The main difference from well-known solutions developed earlier for multidimensional systems [19, 26] is that the vector of the output variables y is available for measurements, however the whole state vector x is not. As shown below (see Section 3.2), due to some principal feature the problem of output-feedback compensation of unknown disturbances for multidimensional systems can not be solved by applying approaches developed for the single input - single output systems.

The problem will be solved in three steps. First, by applying a special observer a parameterized model of the external disturbance in the form of a linear regression with a constant matrix of unknown entries and the physically implementable regressor (measured/calculated by the control and the output) will be obtained. Then an adaptation algorithm for the compensating component of the controller will be derived. Finally, a plant state observer and stabilizing component of the controller will be designed.

3. DISTURBANCE OBSERVER

3.1. Canonical Form of an Exosystem and a Disturbance Parameterization

For the design of a scalar disturbance observer in the output-feedback control, the following lemma is useful since it allows us to express the disturbance δ_i via the *filtered disturbance* $\delta_{fi} = w_i(s)[\delta_i]$, where $w_i(s)$ is the minimum phase asymptotically stable transfer function [19, p. 315].

Lemma 1. *The scalar disturbance δ_i can be represented in the form*

$$\delta_i = \psi_{fi}^\top \xi_{fi} + \epsilon_i, \tag{5}$$

where $\psi_{fi} \in \mathbb{R}^{q_i}$ is the vector of constant unknown parameters, ϵ_i exponentially decays, the regressor $\xi_{fi} \in \mathbb{R}^{q_i}$ is the state vector of the observer

$$\dot{\xi}_{fi} = G_i \xi_{fi} + l_i \delta_{fi}, \tag{6}$$

in which $G_i \in \mathbb{R}^{q_i \times q_i}$ is an arbitrary Hurwitz matrix, $l_i \in \mathbb{R}^{q_i}$ is a constant vector selected so that the pair (G_i, l_i) is controllable.

For the methodological purposes, we represent the proof of Lemma 1 since it is different from the proof reported in [19, p. 315]. Based on its analysis in Section 3.2, we will show the main problems that arise in multidimensional systems.

Proof. Assume the matrices (A_{fi}, b_{fi}, c_{fi}) define the minimal realization of the transfer function $w_i(s)$ and

$$\delta_{fi} = c_{fi}^\top \chi_i, \quad \dot{\chi}_i = A_{fi}\chi_i + b_{fi}\delta_i,$$

where χ_i is the state vector of the filter $w(s)$ with the minimal realization. As it is known [18, p. 87], the forced component χ_i^* of the state vector χ_i for the steady-state model (i.e., without taking into account the exponentially decaying transition component) can be represented in the form $\chi_i^* = M_{\xi i}\xi_i$, where the transformation matrix $M_{\xi i}$ is the solution of the Sylvester equation

$$M_{\xi i}\Gamma_i - A_{fi}M_{\xi i} = b_{fi}h_i^\top. \tag{7}$$

Since A_{fi} is Hurwitz, while the eigenvalues of matrix Γ_i are on the imaginary axis, then this matrix equation has a unique solution $M_{\xi i}$ [18, p. 370]. As a result,

$$\delta_{fi} = \bar{h}_i^\top \xi_i + \bar{\epsilon}_i, \quad \dot{\xi}_i = \Gamma_i \xi_i,$$

where $\bar{h}_i^\top = c_{fi}^\top M_{\xi i}$, and $\bar{\epsilon}_i$ exponentially decays. Since the filter $w_i(s)$ is minimum phase, then the pair $(\bar{h}_i^\top, \Gamma_i)$ is observable. For the forced component ξ_{fi}^* of the state vector ξ_{fi} of the filter (6), the equality $\xi_{fi}^* = M_{fi}\xi_i$ holds, where the transformation matrix M_{fi} is the solution of the Sylvester equation

$$M_{fi}\Gamma_i - G_i M_{fi} = l_i \bar{h}_i^\top. \tag{8}$$

Since the matrices Γ_i and G_i have no common eigenvalues, the pair (G_i, l_i) is fully controllable, the pair $(\Gamma_i, \bar{h}_i)$ is fully observable, then there exist a unique nonsingular matrix M_{fi} satisfying the Sylvester equation (8) [34, p. 240]. Then, $\xi_i = M_{fi}^{-1}\xi_{fi}^*$, and taking into account the second equation of (2) we can write (5) with $\psi_{fi}^\top = h_i^\top M_{fi}^{-1}$.

Remark 4. It worth noting that the solution of matrix equations (7) and (8) is not required for design of the filter (6) and the regression model (5). Instead, it is sufficiently to prove the existence of an invertible matrix M_{fi}.

3.2. Filtered Unmixed Disturbance

Let us introduce the dynamic block in the form

$$\dot{\hat{x}} = A\hat{x} + Bu + L_y(y - C\hat{x}), \tag{9}$$

where $\hat{x} \in \mathbb{R}^n$ is the state vector with arbitrary initial condition $\hat{x}(0)$, and matrix L_y is chosen so that the matrix $A_L = A - L_y C$ is Hurwitz and its eigenvalues do not coincide with invariant zeros of the system (1). Then, for the vector $\varepsilon = x - \hat{x}$ we have

$$\dot{\varepsilon} = A_L \varepsilon + B\delta, \qquad \varepsilon(0) = x(0) - \hat{x}(0). \tag{10}$$

We define the *filtered disturbance* as

$$\delta_f = y - C\hat{x}, \tag{11}$$

for which $\delta_f = C\varepsilon$ holds. Then $\delta_f = W_L(s)[\delta] + \epsilon_f$, where $W_L(s) = C(sI - A_L)^{-1}B$, and $\epsilon_f = Ce^{A_L t}\varepsilon(0)$ exponentially decays.

If $m = 1$, then the regressor of model (5) can be formed using the output of observer (9). Let us investigate if it is possible to extend Lemma 1 to the case of multidimensional systems by replacing the filter with the scalar input (6) with a filter with the vector input

$$\dot{\xi}_f = G\xi_f + L\delta_f,$$

where $G = \mathrm{diag}\{G_i\}$, $L = \mathrm{diag}\{l_i\}$. In the case considered, the vector filtered disturbance δ_f is the output of a model (without taking into account the exponentially decaying term)

$$\delta_f = C\varepsilon, \quad \dot{\varepsilon} = A_L\varepsilon + B\delta,$$

and the Sylvester equation (7) takes the form

$$M_\xi \Gamma - A_L M_\xi = BH, \tag{12}$$

and has the unique solution M_ξ for the given conditions. Then, the filtered vector disturbance δ_f can be represented in the form

$$\delta_f = \bar{H}\xi + \epsilon \quad \dot{\xi} = \Gamma\xi,$$

where ϵ exponentially decays, while the matrix $\bar{H} = CM_\xi$ is not generally block-diagonal. Then equation (8) takes the form

$$M_f\Gamma - GM_f = L\bar{H}. \tag{13}$$

As known, for the case of multidimensional systems (see [34, p. 259] and Example 2.9, p. 54 in [26]) the condition, according to which the matrices Γ_i and G_i have no common eigenvalues, while the pairs (G, L) and $(\Gamma, \bar{H})$ are fully controllable and observable, is necessary conditions, however not sufficient for the existence of a nonsingular solution M_f. In the case of singularity of the matrix M_f, the parametrization $\delta = \Psi\xi_f + \epsilon$, where $\Psi = HM_f^{-1}$, does not exist. Therefore, Lemma 1 can not be extended to the case of a vector disturbance.

A possible way to overcome this problem is to reduce the matrix $\bar{H}$ to a block-diagonal form. Then, the equation (13) can be decomposed into m independent ones of the form (8) ensuring nonsingular solutions.

Let us use the following auxiliary statement.

Lemma 2. *Under assumptions A.1.1–A.1.3, then there exists (possibly not unique) physically implementable asymptotically stable $m \times m$ transfer matrix $Q(s)$ such that the transfer matrix $D(s) = Q(s)W_L(s)$ is the asymptotically stable minimum phase and block-diagonal. One of possible realizations of matrix $Q(s)$ is defined by the equation*

$$Q(s) = \mathrm{adj}W_L(s), \tag{14}$$

where $\mathrm{adj}W_L(s)$ *is the adjoint of a matrix* $W_L(s)$. *In this case,* $D(s) = \mathrm{diag}\left\{\frac{\beta(s)}{\alpha(s)}\right\}$, *where* $\frac{\beta(s)}{\alpha(s)} = \mathrm{det}W_L(s)$.

Proof. Since the plant (1) is fully controllable and observable, the matrices B and C have full rank, and the matrix L is chosen so that the eigenvalues of the matrix A_L do not coincide with invariant zeros of the system (1),[1] then $\det W_L(s) \neq 0$, and the inversion $W_L^{-1}(s) = \frac{\alpha(s)}{\beta(s)}\mathrm{adj}W_L(s)$ exists. At the same time, due to the asymptotic stability of $W_L(s)$ and the minimum phase property of the plant, the polynomials $\beta(s)$ and $\alpha(s)$ are Hurwitz and $\mathrm{adj}W_L(s)$ is asymptotically stable (1) [35, p. 7]. Then, from the equality $W_L^{-1}(s)W_L(s) = \frac{1}{\det W_L(s)}\mathrm{adj}W_L(s)W_L(s) = I$ we have (14). Straightforward calculations of the matrix $Q(s)$ for other block-diagonal forms $D(s)$ shows that this solution is not unique (see the Example in Section 6 "Simulation results").

Let us choose the matrix of a *successive compensator* $Q(s)$ (for example, in form (14)) and make *the filtered unmixed disturbance*

$$\bar{\delta}_f = Q(s)\,[y - C\hat{x}]\,. \tag{15}$$

For $\bar{\delta}_f$ we can write

$$\bar{\delta}_f = \mathrm{diag}\{w_i(s)\}[\delta], \qquad i = 1, \ldots, m, \tag{16}$$

or

$$\bar{\delta}_{fi} = w_i(s)[\delta_i],$$

where $w_i(s)$ are minimum phase asymptotically stable transfer functions (for example, for $Q(s) = \mathrm{adj}W_L(s)$ we have $w_i(s) = \frac{\beta(s)}{\alpha(s)}$, $i = 1, \ldots, m$), and $\bar{\delta}_{fi}$ is ith coordinate $\bar{\delta}_f$.

3.3. Parameterization of the Initial Multidimensional Disturbance

However, the plant (1) is affected by not the filtered unmixed disturbance $\bar{\delta}_f$, but by the initial disturbance δ. Taking into account Lemma 1 and (16), we can formulate the following lemma.

Lemma 3. *Under assumptions A.1.1–A.1.4 and A.2.1–A.2.3, the disturbance δ acting on the plant* (1) *can be represented in the form of the regression model*

$$\delta = \Theta\bar{\xi}_f + \bar{\epsilon}, \tag{17}$$

where $\bar{\epsilon}$ exponentially decays, the regressor $\bar{\xi}_f$ is the state vector of the observer

$$\dot{\bar{\xi}}_f = G\bar{\xi}_f + LQ(s)\,[y - C\hat{x}] \tag{18}$$

with an arbitrary initial condition $\bar{\xi}_f(0)$, the matrices G, L, and Θ are block-diagonal, i.e.,

$$G = \mathrm{diag}\{G_i\}, \quad L = \mathrm{diag}\{l_i\}, \quad \Theta = \mathrm{diag}\{\psi_{fi}^\top\},$$

and $\psi_{fi} \in \mathbb{R}^{q_i}$ are vectors with unknown constant parameters $(i = 1, \ldots, m)$.

Indeed, the dynamic filter (18) with the multidimensional input $\bar{\delta}_f$(15) is splitted into m independent filters (6) with the scalar inputs $\bar{\delta}_i = w_i(s)[\delta_i]$ and the states $\bar{\xi}_{fi}$ what corresponds to the conditions of Lemma 1.

Thus, Lemma 3 reduces the uncertainty of the disturbance δ to the parametric uncertainty of the regression model (17) with the matrix of unknown parameters Θ and the physically realizable regressor $\bar{\xi}_f$. In this case, the plant model (1) can be written as

$$\begin{cases} \dot{x} = Ax + B(u + \Xi\theta + \bar{\epsilon}), \quad x(0), \\ y = Cx, \end{cases} \tag{19}$$

[1] it is known, state feedback – in this case, it is L_yCx – does not affect the zeros of the closed system [34, p. 237].

where $\Xi = \mathrm{diag}\{\bar{\xi}_{fi}^{\top}\} \in \mathbb{R}^{m\times q}$ is the block diagonal matrix regressor that has m vector blocks $\bar{\xi}_{fi}^{\top}$ on the main diagonal, $\theta = [\psi_{f1}^{\top}, \ldots, \psi_{fm}^{\top}]^{\top} \in \mathbb{R}^q$ is the vector of unknown parameters constructed with the diagonal blocks $\psi_{fi}^{\top}$ of the matrix Θ.

Remark 5. As an alternative approach, however not optimal dynamic order of the disturbance observer (and, hence, the number of tuning parameters in the adaptive controller), is the approach, according to which each component δ_{fi} of the filtered disturbance δ_f is considered as the output of a full-dimensional disturbance generator (a similar approach is used in [18, 20, 28]). Indeed, for δ_{fi} we can write

$$\begin{aligned}\delta_{fi} &= \sum_{j=1}^{m} w_{Lij}(s)[\delta_j] = \sum_{j=1}^{m} h_j^{\top} w_{Lij}(s)[\xi_j] = \sum_{j=1}^{m} h_j^{\top} M_{ij}\xi_j + \epsilon_{ij} \\ &= \bar{h}_i^{\top}\xi + \epsilon, \qquad i = 1, \ldots, m,\end{aligned} \tag{20}$$

where $w_{Lij}(s)$ are elements of the transfer matrix $W_L(s)$, M_{ij} is the transformation matrix for the steady-state components, $\bar{h}_i^{\top} = [h_1^{\top} M_{i1}, \ldots, h_m^{\top} M_{im}] \in \mathbb{R}^q$ is the vector of unknown parameters, $\xi \in \mathbb{R}^q$ is the state vector of model (3). The expression (20) motivates us to use a disturbance observer of dimension q in each channel of the control system, which leads to a dynamic order of the observer $m \times q$ in general and to the same number of tuning parameters. At the same time, the proposed model (17) can be obtained using a qth order observer (with a static compensator $Q(s)$ – see Example), the model contains q unknown parameters and, as a result, q adjustable parameters of an adaptive controller.

The analysis of the model (19) motivates the following choice of the control algorithm:

$$u = u_x + u_\delta, \tag{21}$$

where u_x is the stabilizing control component and u_δ is the compensating control component. The next two sections of the paper are devoted to separate design of the control components.

4. DESIGN OF THE ADAPTATION ALGORITHM

Choosing

$$u_\delta = -\Xi\hat{\theta}, \tag{22}$$

where $\hat{\theta}$ is the vector of adjustable parameters, and substituting (21) and (22) into (19), we derive the model of control error

$$\begin{cases} \dot{x} = Ax + B(u_x + \Xi\tilde{\theta} + \bar{\epsilon}), \quad x(0), \\ y = Cx \end{cases} \tag{23}$$

with the vector of parametric errors $\tilde{\theta} = \theta - \hat{\theta}$.

Since the state vector x is not measurable, and the matrix A can be unstable, for design of an adaptation algorithm generating $\hat{\theta}$ we will form a static regression model defined by the following statement.

Lemma 4. *Let us introduce the extended error*

$$\bar{y} = y - C\hat{x} - \Xi_f\hat{\theta}, \tag{24}$$

where the vector $\hat{x}$ *is generated by the filter* (9), $\Xi_f \in \mathbb{R}^{m\times qm}$ *is the matrix regressor of the form*

$$\Xi_f = \begin{bmatrix} w_{L\,11}(s)\left[\bar{\xi}_{f1}^{\top}(t)\right] & \dots & w_{L\,1m}(s)\left[\bar{\xi}_{fm}^{\top}(t)\right] \\ \vdots & \ddots & \vdots \\ w_{L\,m1}(s)\left[\bar{\xi}_{f1}^{\top}(t)\right] & \dots & w_{L\,mm}(s)\left[\bar{\xi}_{fm}^{\top}(t)\right] \end{bmatrix}, \tag{25}$$

and $w_{L\,ij}(s)$ *are the entries of the asymptotically stable transfer function* $W_L(s) = C(sI - A_L)^{-1}B$ $(i = 1,\dots,m,\ j = 1,\dots,m)$. *Then, the following equality holds for the signal* $\bar{y}$:

$$\bar{y} = \Xi_f\tilde{\theta} + \bar{\epsilon}_f, \tag{26}$$

where $\bar{\epsilon}_f$ *exponentially decays.*

The proof of Lemma 4 is given in Appendix.

Model (26) is well known in the modern theory of adaptive systems [36, 37] and identification [38] and allows using a wide class of standard adaptation algorithms such as:

— *gradient adaptation algorithm* [19, 36, 37]

$$\dot{\hat{\theta}} = \gamma\Xi_f^{\top}\bar{y}; \tag{27}$$

— *algorithm with improved parametric convergence* [19, 39]

$$\dot{\hat{\theta}} = \gamma\left(d(s)\left[\Xi_f^{\top}\bar{y}\right] - \Omega\hat{\theta}\right), \tag{28}$$

where $\gamma > 0$ is the parameter of adaptation, $\Omega = d(s)[\Xi_f^{\top}\Xi_f]$ is the extended matrix regressor, $d(s)$ is a minimum phase asymptotically stable positive transfer function with real roots of the characteristic polynomial and the unit static gain ($d(0) = 1$). In the simplest case, $d(s)$ can be selected as the first order block. The convergence properties of the adaptation algorithms (27) and (28) are defined by the following statement with the proof given in Appendix.

Lemma 5. *If assumptions A.1.1–A.1.4 and A.2.1–A.2.3 hold, then*

L.5.1. *adaptation algorithms* (27) *and* (28) *ensure the boundedness of* $\bar{y}$ *and* $\hat{\theta}$, *and also the asymptotic tendency of* $|\Xi(t)\tilde{\theta}(t)| \to 0$ *as* $t \to \infty$;

L.5.2. *if* $\lambda(t) \notin \mathcal{L}_1$, *where* $\lambda(t)$ *is the minimum eigenvalue of the matrix* $\Omega(t)$, *then adaptation algorithm* (28) *provides asymptotic convergence* $|\tilde{\theta}(t)| \to 0$ *as* $t \to \infty$ *in addition to property L.5.1;*

L.5.3. *if the regressor* ξ *satisfies the condition of persistent excitation* (*in the sense of definition 4.3.1 from* [36] *or definition 3.4 from* [19]), *then adaptation algorithms* (27) *and* (28) *in addition to property L.5.1 provide exponential convergence* $|\tilde{\theta}(t)| \to 0$ *as* $t \to \infty$.

5. DESIGN OF A STABILIZING COMPONENT AND PROPERTIES OF A CLOSED-LOOP SYSTEM

Since the vector x is not directly measurable, we use a state observer of the form

$$\dot{\bar{x}} = A\bar{x} + Bu_x + L_y(y - C\bar{x}), \tag{29}$$

where $\bar{x}$ is the state vector of the observer with an arbitrary initial condition $\bar{x}(0)$. Let us introduce the vector $\tilde{x} = x - \bar{x}$. Subtracting (29) from (23), we obtain

$$\dot{\tilde{x}} = A_L\tilde{x} + B(\Xi\tilde{\theta} + \bar{\epsilon}).$$

In relation to property L.5.1 of Lemma 5 we have $|\Xi(t)\tilde{\theta}(t)| \to 0$, therefore, $|\tilde{x}(t)| \to 0$ as $t \to \infty$.

A stabilizing control can be represented as

$$u_x = -K\bar{x}, \tag{30}$$

where the feedback matrix is such that the matrix $A_K = A - BK$ is Hurwitz. Substituting (30) into (23), we obtain the model of a closed-loop system.

$$\dot{x} = A_K x + B(\Xi\tilde{\theta} - K\tilde{x} + \bar{\epsilon}).$$

Since $|\tilde{x}(t)| \to 0$ for $t \to \infty$, we immediately get that $|x(t)| \to 0$ as $t \to \infty$, which means that control objective (4) is achieved. Thus, the following statement was proved.

Theorem 1. *Under assumptions A.1.1–A.1.4 and A.2.1–A.2.3, the control law* (21) *containing the compensating component* (22)*, the stabilizing component* (30)*, the disturbance observer* (9), (18)*, the adaptation algorithm* (27) *or* (28) *and the state observer* (29)*, being applied to the plant* (1)*, ensures the boundedness of all signals and the achievement of control objective* (4)*.*

6. EXAMPLE AND THE SIMULATION RESULTS

Let us consider an unstable control plant (1) with matrices

$$A = \begin{bmatrix} 4 & -6 & 0 \\ 4.5 & -7 & 0 \\ 12 & -20 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 2 & 0 \\ 4 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} -2 & 4 & -1 \\ 3 & -5 & 1 \end{bmatrix}$$

and the transfer matrix

$$W(s) = C(sI - A)^{-1}B = \begin{bmatrix} 0 & \dfrac{-1}{s-1} \\ \dfrac{1}{s^2+3s-1} & \dfrac{1}{s-1} \end{bmatrix}.$$

In this case, the plant has no invariant zeros and hence is minimum phase.

We assume that $\delta = [\delta_1,\ \delta_2]^\top = [\sin t, 1]^\top$ is unmeasurable and a priori unknown disturbance, the first component of which can be considered as the output of the exosystem (2) of the second order, while the second component can be considered as the output of exosystem (2) of the first order. Then, we can write

$$\Gamma = \begin{bmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} h_1^\top & 0 \\ 0 & h_2^\top \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

If we choose

$$L_y = \begin{bmatrix} 6 & 6 \\ 6 & 6 \\ 10 & 12 \end{bmatrix},$$

then,

$$A_L = \begin{bmatrix} -2 & 0 & 0 \\ -1.5 & -1 & 0 \\ -4 & 0 & -1 \end{bmatrix}, \quad W_L(s) = \begin{bmatrix} 0 & \dfrac{-1}{s+1} \\ \dfrac{1}{(s+1)(s+2)} & \dfrac{1}{s+1} \end{bmatrix}.$$

Finally, for the disturbance observer design we choose

$$G = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -6 & -5 & 0 \\ 0 & 0 & -4 \end{bmatrix}, \quad L = \begin{bmatrix} l_1 & 0 \\ 0 & l_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 6 & 0 \\ 0 & 4 \end{bmatrix}.$$

For the methodological purposes, we calculate the matrices M_ξ, $\bar{H}$, and M_f that are solutions of the equations (12) and (13), respectively. Then, we obtain nonblock-diagonal matrix

$$\bar{H} = CM_\xi = \begin{bmatrix} 0 & 0 & -1 \\ 0.1 & -0.3 & 1 \end{bmatrix}$$

and the singular matrix

$$M_f = \begin{bmatrix} 0 & 0 & -0.33 \\ 0 & 0 & 0 \\ -0.04 & -0.28 & 1 \end{bmatrix}.$$

It is shown that the direct extension of Lemma 1 to the case of a vector disturbance is impossible without special methods.

According to the proposed approach, we calculate the transfer matrix of the successive compensator

$$Q(s) = \mathrm{adj}W_L(s) = \begin{bmatrix} \dfrac{1}{s+1} & \dfrac{1}{s+1} \\ \dfrac{-1}{(s+1)(s+2)} & 0 \end{bmatrix} \tag{31}$$

and form filtered unmixed disturbance (15). Then, we can show that the following model is valid for $\bar{\delta}_f$:

$$\bar{\delta}_f = \begin{bmatrix} \dfrac{1}{(s+1)^2(s+2)} & 0 \\ 0 & \dfrac{1}{(s+1)^2(s+2)} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{(s+1)^2(s+2)}[\delta_1] \\ \dfrac{1}{(s+1)^2(s+2)}[\delta_2] \end{bmatrix}.$$

However, according to Lemma 2, the compensator (31) that leads the disturbance to an unmixed form is not unique. Indeed, in the example, the simple successive compensator

$$Q(s) = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \tag{32}$$

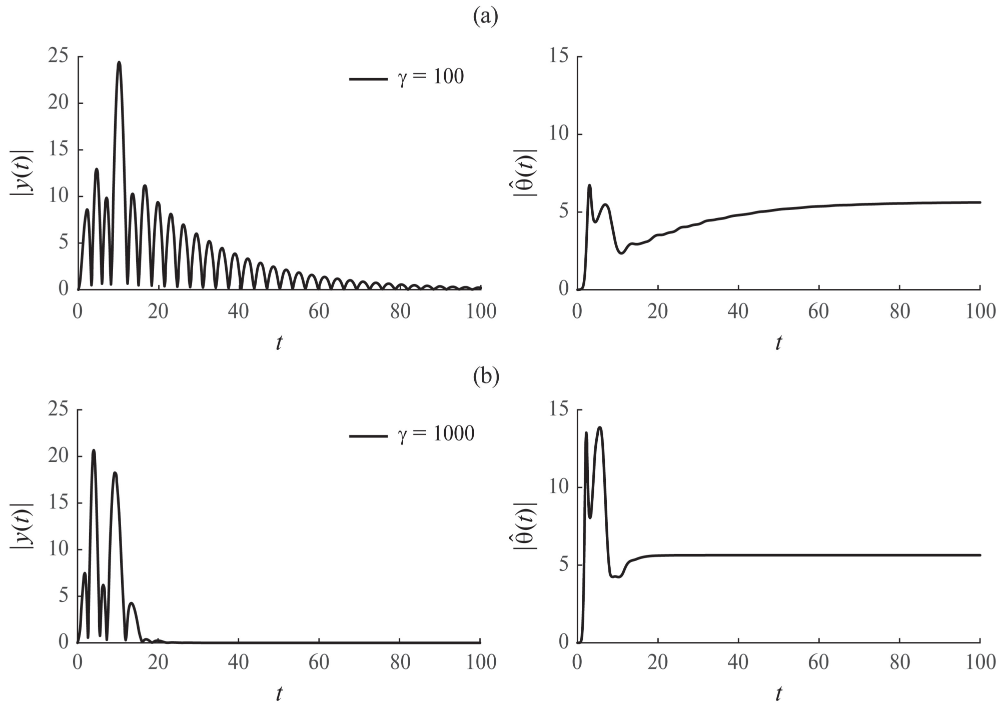
gives the following model of unmixed filtered disturbance:

$$\bar{\delta}_f = \begin{bmatrix} \dfrac{1}{(s+1)(s+2)} & 0 \\ 0 & \dfrac{1}{s+1} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{(s+1)(s+2)}[\delta_1] \\ \dfrac{1}{s+1}[\delta_2] \end{bmatrix}.$$

For simulation, the successive compensator (32) is applied.

Simulation results of the closed-loop system with the adaptation algorithm with improved convergence (28) for $d(s) = \frac{1}{s+1}$ and two different of γ, the stabilizing control (30) with

$$K = \begin{bmatrix} 25.0 & -0.91 & -8.38 \\ 36.25 & -4.83 & -10.67 \end{bmatrix},$$



Transients in the adaptive system closed by the adaptation algorithm with improved parametric convergence for: (a) $\gamma = 100$; (b) $\gamma = 1000$.

for zero initial conditions of the plant and the control algorithm are shown in the Figure. The simulation results demonstrate the achievement of control objective (4) in the presence of external previously unknown disturbance and the opportunity to accelerate the process of tuning controller by increasing the adaptation parameter γ.

7. CONCLUSION

The paper presents the solution to the problem of the output-feedback adaptive compensation of external unknown deterministic disturbances for multidimensional linear systems. The proposed solution is based on the adaptive implementation of the internal model principle and design of a special observer, which makes it possible to obtain an unmixed parametrization of the disturbance. The presented approach allows us to design an adaptive controller with a number of tuning parameters equaled to the number of unknown coefficients of the characteristic polynomial of the disturbance model, thereby, reducing the computational complexity of the algorithm compared to known solutions.

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APPENDIX

Proof of Lemma 4. We rewrite (25) in the state-space form

$$\begin{cases} \dot{\Xi}_x = A_L \Xi_x + B\Xi, \\ \Xi_f = C\Xi_x. \end{cases} \tag{A.1}$$

Then, we introduce an intermediate variable

$$\epsilon_x = x - \hat{x} - \Xi_x\theta \tag{A.2}$$

and calculate its time derivative according to (19), (9) and (A.1):

$$\dot{\epsilon}_x = x - \hat{x} - \Xi_x\theta = Ax + Bu + B\Xi\theta + \bar{\epsilon} - A\hat{x} - Bu - L_y(Cx - C\hat{x}) - A_L\Xi_x\theta - B\Xi\theta = A_L\epsilon_x + \bar{\epsilon}.$$

Due to the Hurwitz property of A_L, the value $\epsilon_x(t)$ exponentially decays.

Then $x - \hat{x} = \Xi_x\theta + \epsilon_x$ or

$$y - C\hat{x} = \Xi_f\theta + \bar{\epsilon}_f, \tag{A.3}$$

where $\bar{\epsilon}_f = C\epsilon_x$. Substituting (A.3) into (24), we obtain (26).

Proof of Lemma 5. We choose the Lyapunov function

$$V = \frac{1}{2\gamma}\tilde{\theta}^\top\tilde{\theta} + \int\limits_t^\infty \bar{\epsilon}_f^\top(\tau)\bar{\epsilon}_f(\tau)d\tau \tag{A.4}$$

and calculate its time derivative in view of (26), the equality $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$ and the algorithms (27) and (28):

1. The algorithm (27).

$$\dot{V} = -\tilde{\theta}^\top\Xi_f^\top\Xi_f\tilde{\theta} - \tilde{\theta}^\top\Xi_f^\top\bar{\epsilon}_f - \bar{\epsilon}_f^\top\bar{\epsilon}_f = -\frac{1}{2}|\Xi_f\tilde{\theta}|^2 - \frac{1}{2}|\bar{\epsilon}_f| - \frac{1}{2}\left(|\Xi_f\tilde{\theta}| + |\bar{\epsilon}_f|\right)^2 \leqslant -\frac{1}{2}|\Xi_f\tilde{\theta}|^2.$$

From the last expression, it follows that the signals $\hat{\theta}$ and $|\Xi_f\tilde{\theta}| \in \mathcal{L}_2$ are bounded. Due to the boundedness of Ξ and the stability of the filter $W_L(s)$, the output of linear regression (26) $\bar{y}$, and the derivative $\dot{\hat{\theta}}$ are bounded. Since the function $|\Xi_f(t)\tilde{\theta}(t)| \in \mathcal{L}_2$ and it is continuous and bounded, then $|\Xi_f(t)\tilde{\theta}(t)|$, $\dot{\hat{\theta}}(t) \to 0$ as $t \to \infty$.

According to the swapping lemma applied to multichannel systems (see [19, Section D.2]), we have

$$W_L(s)\left[\Xi\tilde{\theta}\right] = \Xi_f\tilde{\theta} + W_L(s)\left[\Xi_f\dot{\hat{\theta}}\right], \tag{A.5}$$

where Ξ_f is the output of the filter (A.1) from Appendix. From the last expression follows convergence $W_L(s)\left[\Xi(t)\tilde{\theta}(t)\right] \to 0$ as $t \to \infty$ due to the boundedness of $\Xi(t)$, stability of $W_L(s)$, and the convergence $|\Xi_f(t)\tilde{\theta}(t)|$, $\dot{\hat{\theta}}(t) \to 0$. Since $W_L(s)$ is nonsingular and minimum phase, and $\dot{\Xi}$, $\dot{\hat{\theta}}$ are bounded, then $|\Xi(t)\tilde{\theta}(t)| \to 0$ as $t \to \infty$.

2. The algorithm (28). Let us analyze the derivative of the function V taking into account the Cauchy–Bunyakovsky inequality:

$$\dot{V} = -\tilde{\theta}^\top\Omega\tilde{\theta} - \tilde{\theta}^\top d(s)\left[\Xi_f^\top\bar{\epsilon}_f\right] - \bar{\epsilon}_f^\top\bar{\epsilon}_f = -\tilde{\theta}^\top\Omega\tilde{\theta} - \tilde{\theta}^\top(t)\int\limits_0^t h(t-\tau)\Xi_f^\top(\tau)\bar{\epsilon}_f(\tau)d\tau - |\bar{\epsilon}_f|^2$$

$$\leqslant -\tilde{\theta}^\top\Omega\tilde{\theta} + \left(\int\limits_0^t h(t-\tau)\left(\tilde{\theta}(t)\Xi_f^\top(\tau)\right)^2 d\tau\right)^{\frac{1}{2}}\left(\int\limits_0^t h(t-\tau)\bar{\epsilon}_f^2(\tau)d\tau\right)^{\frac{1}{2}} - |\bar{\epsilon}_f|^2,$$

where $h(t-\tau) \geqslant 0$ is the impulse response of the filter $d(s)$. Taking into account, that

$$\int_0^t h(t-\tau)\left(\tilde{\theta}(t)\Xi_f^\top(\tau)\right)^2 d\tau \leqslant c_d \tilde{\theta}^\top(t)\Omega(t)\tilde{\theta}(t), \quad \int_0^t h(t-\tau)\bar{\epsilon}_f^2(\tau)d\tau \leqslant c_d|\bar{\epsilon}_f|^2,$$

where $c_d = |d(s)|_\infty = 1$, we can continue the analysis $\dot{V}$:

$$\dot{V} \leqslant -\tilde{\theta}^\top\Omega\tilde{\theta} + \left(\tilde{\theta}^\top\Omega\tilde{\theta}\right)^{\frac{1}{2}}|\bar{\epsilon}_f| - |\bar{\epsilon}_f|^2 \leqslant -\frac{1}{2}\tilde{\theta}^\top\Omega\tilde{\theta}. \tag{A.6}$$

From the last expression follows that the signals $\hat{\theta}$ and $|\Omega^{\frac{1}{2}}\tilde{\theta}| \in \mathcal{L}_2$ are bounded. Therefore, due to the stability of the filters $W_L(s)$ and $d(s)$, the functions $\Omega(t)$ and $\dot{\Omega}$ are bounded. Therefore, $|\Omega\tilde{\theta}| \in \mathcal{L}_2$ and $|\Omega(t)\tilde{\theta}(t)| \to 0$ as $t \to \infty$. Since $\dot{\tilde{\theta}}(t) \to 0$, then (see the Swapping Lemma in [40]) $d(s)\left[\Xi_f^\top(t)\Xi_f(t)\tilde{\theta}(t)\right] \to 0$ as $t \to \infty$. Since $\dot{\Xi}_f$ and $\dot{\tilde{\theta}}$ are bounded, then from the convergence $d(s)\left[\Xi_f^\top(t)\Xi_f(t)\tilde{\theta}(t)\right] \to 0$ it follows that $\Xi_f(t)\tilde{\theta}(t) \to 0$ as $t \to \infty$. Taking into account the arguments mentioned above for the proof of property L.5.1 for the algorithm (27), from the convergence $\Xi_f(t)\tilde{\theta}(t) \to 0$ we have the convergence $\Xi(t)\tilde{\theta}(t) \to 0$ as $t \to \infty$.

Property L.5.1 is proved.

To prove property L.5.2, we proceed the analysis of the derivative (A.6) taking into account (A.4):

$$\dot{V} \leqslant -\frac{1}{2}\tilde{\theta}^\top\Omega\tilde{\theta} \leqslant -\frac{1}{2}\lambda(t)\tilde{\theta}^\top\tilde{\theta} = -\gamma V + \gamma\int_t^\infty \bar{\epsilon}_f^\top(\tau)\bar{\epsilon}_f(\tau)d\tau.$$

Solving the obtained differential inequality, we have

$$V(t) \leqslant \mathrm{e}^{-\gamma\int_0^t \lambda(\tau_1)d\tau_1} V(0) + \gamma\int_0^t \mathrm{e}^{-\gamma\int_{\tau_3}^t \lambda(\tau_2)d\tau_2} \int_{\tau_3}^\infty \bar{\epsilon}_f^\top(\tau_1)\bar{\epsilon}_f(\tau_1)d\tau_1 d\tau_3,$$

from which it follows property L.5.2.

The proof of property L.5.3 for the algorithm (27) can be found in [37, Section 2.8]. The proof of property L.5.3 for the algorithm (28) can be found in [19, Section 3.2.3].

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